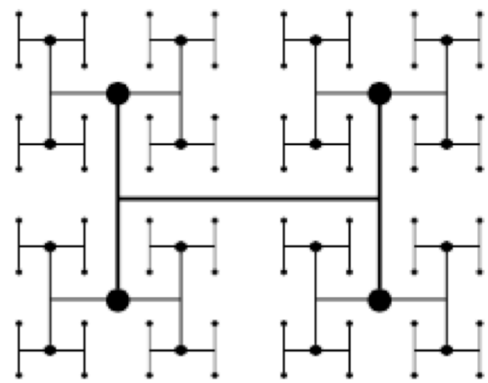
Fractals and Exponents

Overview

In this exercise, students use fractals to learn about exponents, specifically with the bases of 2, 3, 4 and 10. Students fill in tables, graph their results and formulate equations to match the graph. In the end, students compare relative scales of natural objects using exponent rules and make a few measurements in the classroom to compare relative sizes.



Appropriate for: grades 4 – 8, college and adults

Objectives

- To create a complex pattern by repeating a simple process
- To identify and describe shapes
- To analyze, compare, create and compose shapes
- To generate and analyze patterns
- To model with mathematics
- To attend to precision
- To draw, construct and describe geometrical figures and describe the relationships between them
- To generate and analyze patterns
- To use operations to explain patterns in arithmetic
- To write and interpret numerical expressions

Materials

- Fractals and exponents worksheet
- Pencil
- Rulers, meter stick and 50 m tape measure
- Calculator (optional)

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Common Core Standards for Mathematics

Code	Standard	Grade	Code	Standard	Grade
OA	Operations and Algebraic Thinking	4, 5	NS	Number System	6, 7
NBT	Numbers and Operations in Base Ten	4	RP	Ratios and Proportional Relationships	6, 7
NF	Numbers and Operations – Fractions	4	EE	Expressions and Equations	6 – 8
MD	Measurement and Data	4, 5	F	Functions	8
G	Geometry	5			

HS: Numbers (RN), Algebra (SSE, CED), Functions (BF, LE)

Common Core Standards for English Language Arts

Code	Standard	Grades K – 5	Grades 6 – 8	Grades 9 – 12
RL	Reading: Literature	1, 4, 7, 10	1, 4, 7, 10	1, 4, 10
RI	Reading: Informational Text	1, 3, 4, 7, 10	1, 3, 4, 7, 10	1, 3, 4, 10
FS	Foundational Skills	1, 2, 3 for grades K – 1; 3 and 4 for grades 2 – 5	None available	None available
W	Writing	2, 3, 8; 4 for grades 3 – 5	2, 3, 4	2, 3, 4, 9
SL	Speaking and Listening	1, 2, 3, 4, 5, 6	1, 2, 3, 4, 5, 6	1, 2, 3, 4, 5, 6
L	Language	1, 4, 6; 3 for grades 2 – 5	1, 3, 4, 6	1, 3, 4, 6
RST	Science and Technical Subjects	None available	1, 3, 4, 6, 7, 10	2, 3, 4, 6, 7, 10

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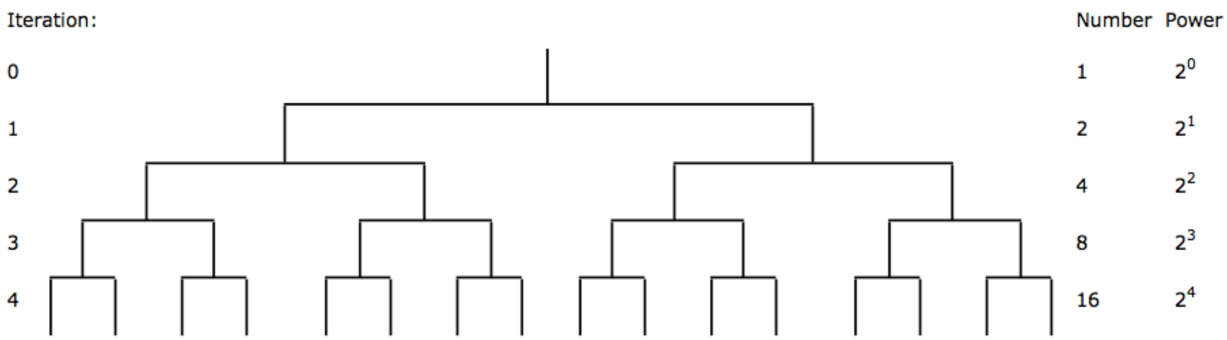
Instructions

The **Powers of 2** is a very compact and useful way to describe numbers in this pattern. It represents how many times we have to multiply 2 to reach the number. The binary branching pattern (shown below) is one of the most basic fractal patterns. We can find examples of this type of pattern in nature, including trees, river networks, blood vessels and lightning bolts. This pattern also occurs in human structures such as family trees and sports tournaments.



At the beginning (iteration 0), there is one branch. After the first iteration, it splits into two branches. At the second iteration, the pattern doubles and forms four branches. This pattern continues and continues. On the left, you see the number of iterations. On the right, the number of branches are shown as well as the mathematical notation as a power of two.

0 1 2 3 4 5

Less commonly used, but still interesting, are the **Powers of 3**. We can observe this pattern in the Sierpinski Triangle sequence shown above. At each step, there are three times as many triangles as the step before.

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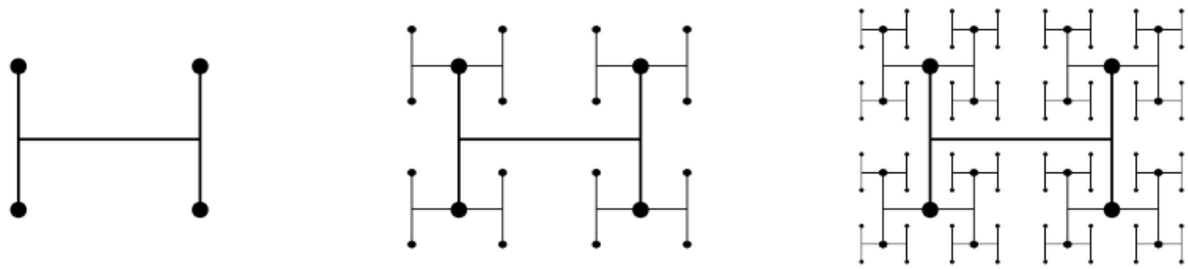
Fractals and Exponents



Iteration 1 4 Dots | Iteration 2 16 Dots | Iteration 3 ? Dots ?

The "H" Fractal illustrates the progression of the **Powers of 4**. At each step, there are four times as many of the end-point dots. We can describe these numbers as $4^1 = 4$, $4^2 = 4 \times 4 = 16$, $4^3 = 4 \times 4 \times 4$

The powers of 2, 3 and 4 create interesting sequences of numbers like {2, 4, 8, 16, 32, 64...}, {3, 9, 27, 81, 243, 729...} and {4, 16, 64, 256, 1024, 4096...}

The **Powers of 10** are much simpler and easier to use.

1	10	10^1	10
2	10 x 10	10^2	100
3	10 x 10 x 10	10^3	1,000
4	10 x 10 x 10 x 10	10^4	10,000
5	10 x 10 x 10 x 10 x 10	10^5	100,000
6	10 x 10 x 10 x 10 x 10 x 10	10^6	1,000,000

Each iteration (multiplication by 10) adds another zero to the number. An extremely useful trick lets you multiply two powers of ten by simply adding the exponents. So for example, $10^3 \times 10^3 = 10^{(3+3)} = 10^6$. It is much easier and more compact to write using exponents than writing (10 x 10 x 10) x (10 x 10 x 10) or 1,000 x 1,000 = 1,000,000.

To divide two numbers written this way, you simply subtract their exponents. For example, one billion / one million = 1,000,000,000 / 1,000,000 = $10^9 - 10^6 = 10^{(9-6)} = 10^3$ = 1,000

These rules work for negative exponents too. Negative exponents are just an inverse or a fraction of the number, so for example $10^2 = 100$, and $10^{-2} = 1/100$ or 0.01.

Since fractals in nature span a huge range of sizes, we need to use very large or small numbers to describe them. It is very convenient to use exponents to describe and compare the scales of fractals.

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