

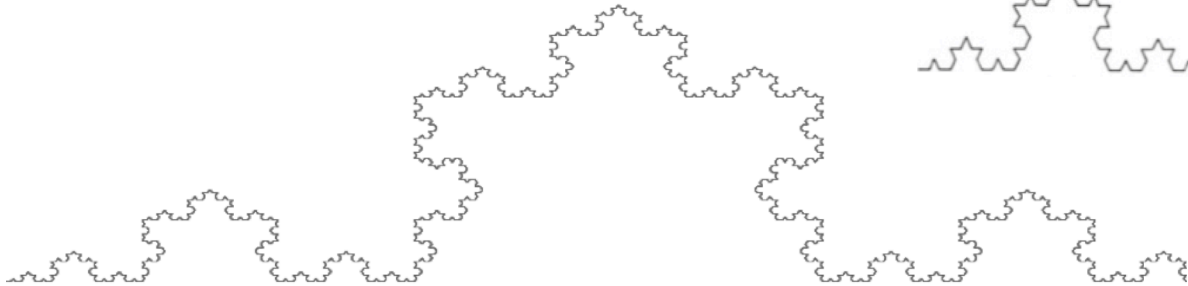
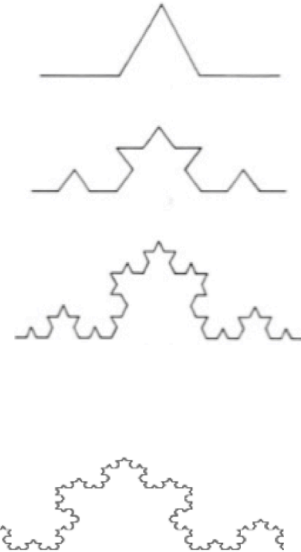


The Koch Curve and Coastlines



Overview

In this exercise, students learn to transform a simple shape into a complex fractal curve by repeating the same operation at smaller and smaller scales and track changes to the perimeter at each step. The resulting shape is highly complex, has a large perimeter and is roughly similar to natural fractals like coastlines.



In the coastline exercise, students learn that the act of measuring the length of a natural shape such as a coastline depends on the scale of the ruler with which they measure it. This insight leads to the concepts of self-similarity and fractal dimension.

Appropriate for: grades 3 – 12 , college and adults

Objectives

- To create a complex pattern by repeating a simple process
- To identify and describe shapes
- To analyze, compare, create, and compose shapes
- To generate and analyze patterns
- To model with mathematics
- To classify shapes by properties of their lines and angles
- To attend to precision
- To draw, construct and describe geometrical figures and describe the relationships between them
- To report data in tables and graph data to see trends

Materials

- Pencil
- Koch Curve and Coastlines worksheet
- Scissors

Fractals are SMART: Science, Math & Art!

www.FractalFoundation.org

Copyright 2015 Fractal Foundation, all rights reserved



The Koch Curve and Coastlines



Common Core Standards for Mathematics

Code	Standard	Grade	Code	Standard	Grade
OA	Operations and Algebraic Thinking	3 – 5	RP	Ratios and Proportional Relationships	6, 7
MD	Measurement and Data	3 – 5	SP	Statistics and Probability	6
G	Geometry	3, 5, 6	EE	Expressions and Equations	7
NS	Number System	6, 7	F	Functions	8

HS: Numbers (Q), Algebra (CED), Functions (LE), Modeling, Geometry (SRT, MG)

Common Core Standards for English Language Arts

Code	Standard	Grades K – 5	Grades 6 – 8	Grades 9 – 12
RL	Reading: Literature	1, 4, 7, 10	1, 4, 7, 10	1, 4, 10
RI	Reading: Informational Text	1, 3, 4, 7, 10	1, 3, 4, 7, 10	1, 3, 4, 10
FS	Foundational Skills	1, 2, 3 for grades K – 1; 3 and 4 for grades 2 – 5	None available	None available
W	Writing	2, 3, 8; 4 for grades 3 – 5	2, 3, 4	2, 3, 4, 9
SL	Speaking and Listening	1, 2, 3, 4, 5, 6	1, 2, 3, 4, 5, 6	1, 2, 3, 4, 5, 6
L	Language	1, 4, 6; 3 for grades 2 – 5	1, 3, 4, 6	1, 3, 4, 6
RST	Science and Technical Subjects	None available	1, 3, 4, 6, 7, 10	2, 3, 4, 6, 7, 10



The Koch Curve and Coastlines



Instructions

The Koch Curve is a fractal that starts with a simple pattern made from a line that is divided into three equal parts.

First, erase the middle segment and replace it with an upside down "V" shape. Now the pattern is made up of four line segments.

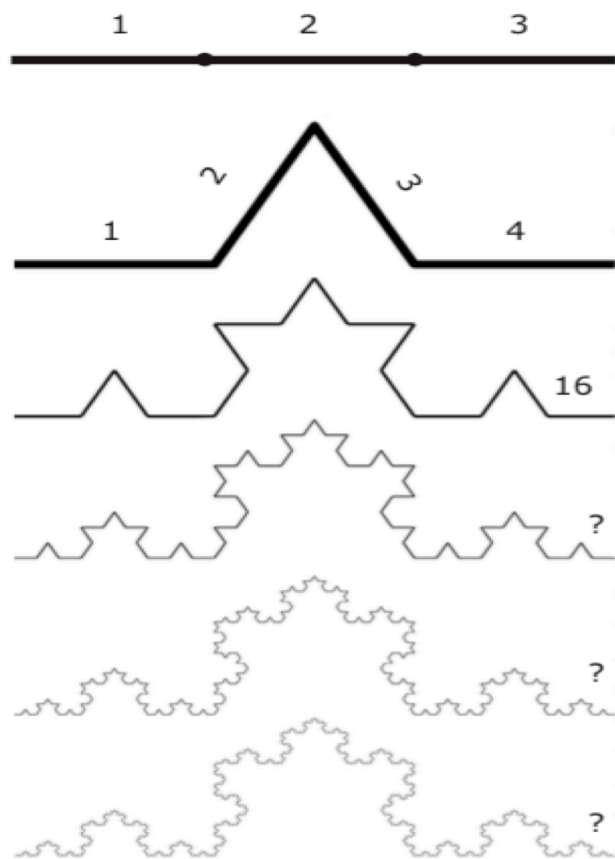
Next, we repeat the process. Each of those four lines is divided in thirds and the middle segment is replaced with a "V".

There are now 4×4 or 16 line segments. Remember: fractals are never-ending patterns made by repeating the same idea over again. Next, we replace each of the 16 line segments with the same pattern again.

Each time we do this, the curve gets more jagged and complicated, and its length – or perimeter – gets bigger.

Perimeter = is the distance around a two dimensional shape, or the measurement of the distance around something

Eventually, the pattern starts to look like a fractal in nature like a coastline, or the edge of a snowflake.





The Koch Curve and Coastlines



Coastline exercise

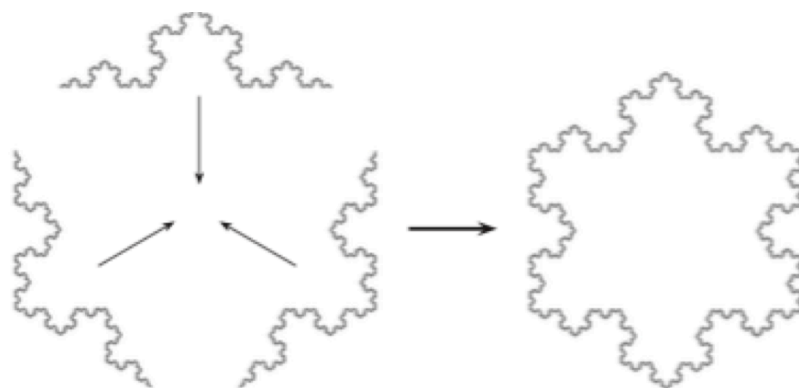
The field of fractal geometry began by asking the deceptively simple question: *How long is the coastline of Great Britain?* The answer is: it depends how closely you measure it! With a very small ruler, the length (or perimeter) is very large, as more and more details appear at small scales. Imagine measuring every rock, pebble and grain of sand on the beach!

Coastlines are usually not smooth, straight lines, but instead they have lots of similar detail at different sizes. Coastlines are fractals! *Why are coastlines fractals?* Coastlines are fractals because they are formed by simple, repetitive processes over thousands or millions of years. The crashing of waves slowly erodes the coastline. So does the rising and falling of the tides. Giant storms also erode the coastline, leaving behind fractal patterns.



In this exercise, you will measure the length of the coastline using rulers of three different lengths and graph your results. First, cut out the paper ruler. Use the long edge (10 cm length) of the ruler first, then fold it in half to measure the perimeter at a 5 cm scale, and finally use the edge to measure the perimeter at a 2.5 cm scale.

What do snowflakes and Koch Curves have in common? If you take three copies of the Koch Curve, rotate them and combine them as shown below, you end up with a six-fold symmetric object that looks like a snowflake! Snowflakes grow by expanding outward from the center while continuously branching. It's not *exactly* the same process as the Koch Curve, but they are both made by a repeating process. Like all fractals, complex shapes come from simple repetition.



Fractals are SMART: Science, Math & Art!

www.FractalFoundation.org

Copyright 2015 Fractal Foundation, all rights reserved