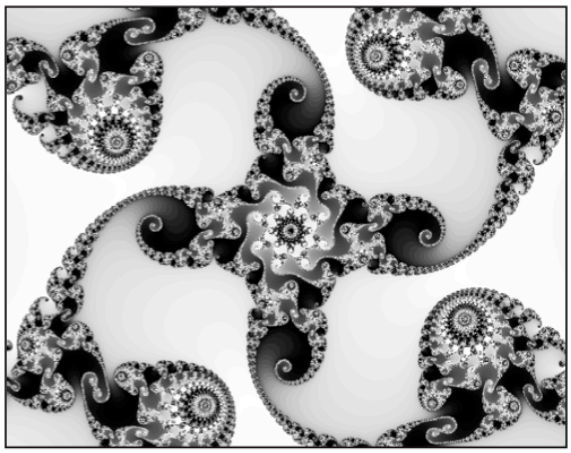
Fractals and XaoS

Overview

XaoS, free fractal software, allows us to interact with fractals in a compelling and artistic way. This exercise guides users of XaoS in downloading and navigating the software, exploring mathematical patterns in fractals and creating beautiful personalized art work. If you have access to a projector, you can take your students on a tour of the Mandelbrot set before letting them explore it themselves.

XaoS is the program used to create fractal art for students to submit to the annual Fractal Challenge. You can use it to create your own greeting cards… basically for anything you want a design on!

Appropriate for: K-12, college and adults



Objectives

- To understand how to navigate XaoS computer software
- To investigate the mathematical patterns in fractals
- To analyze, compare, create, and compose shapes
- To generate and analyze patterns
- To create individualized fractal art

Materials

- Computer
- XaoS downloaded or internet connection to download (http://fractalfoundation.org/resources/fractal-software/)
- Protractors (optional)

Fractals and XaoS

Common Core Standards for Mathematics

Code	Standard	Grade	Code	Standard	Grade
CC	Counting and Cardinality	K	G	Geometry	K – 4
NBT	Number and Operations in Base Ten	K, 1	RP	Ratios and Proportional Relationships	6, 7
OA	Operations and Algebraic Thinking	K – 5	EE	Expressions and Equations	6, 7
MD	Measurement and Data	K	F	Functions	8

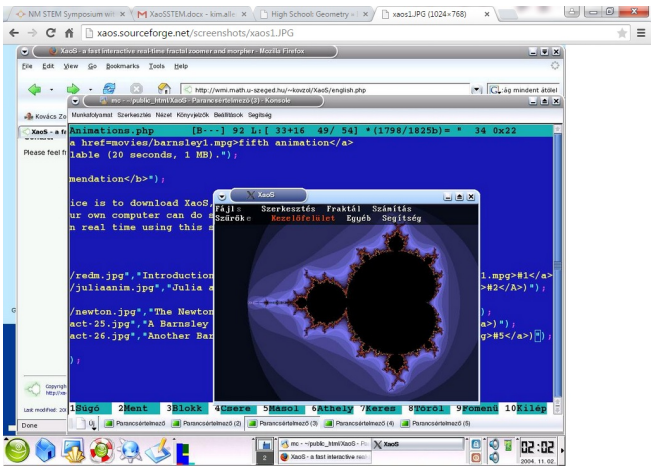
Common Core Standards for English Language Arts

Code	Standard	Grades K – 5	Grades 6 – 8	Grades 9 – 12
RL	Reading: Literature	1, 4, 7, 10	1, 4, 7, 10	1, 4, 10
RI	Reading: Informational Text	1, 3, 4, 7, 10	1, 3, 4, 7, 10	1, 3, 4, 10
FS	Foundational Skills	1, 2, 3 for grades K – 1; 3 and 4 for grades 2 – 5	None available	None available
W	Writing	2, 3, 8; 4 for grades 3 – 5	2, 3, 4	2, 3, 4, 9
SL	Speaking and Listening	1, 2, 3, 4, 5, 6	1, 2, 3, 4, 5, 6	1, 2, 3, 4, 5, 6
L	Language	1, 4, 6; 3 for grades 2 – 5	1, 3, 4, 6	1, 3, 4, 6
RST	Science and Technical Subjects	None available	1, 3, 4, 6, 7, 10	2, 3, 4, 6, 7, 10

Fractals and XaoS

Instructions

First, download the correct version of the program for your computer (Mac or PC). Follow the instructions when promted.



Note: For some school computers, installation is complicated by restricted permissions. You may need an IT staff or administrator for help.

Setting defaults (only necessary the first time): From the "Calculation" menu, select "Iterations" and set the value to 2,000. From the "Filters" menu, select "Palette Emulator". From the "File" menu, select "Save Configuration" to save these settings.

When you first run the program, it opens with an image of the Mandelbrot set. To navigate, point the mouse and click! Hold the left top part the mouse down to zoom in. Hold the right top part of the mouse down to zoom out. On a laptop, holding one finger down zooms in and two fingers down zoom out.

What is a Mandelbrot set? It is the set of complex numbers 'c' for which the sequence stays finite.

To pan the image around, use both buttons together for a PC, or <shift> click on the Mac.

Color palettes are randomly generated, and can be changed with the "P" key. To enable or disable color cycling, use "Y". There are many filters and effects to explore from the menus.

XaoS can create many different fractal types, which can be accessed by using the number keys. Keys 1 to 5 are Mandelbrot sets with various powers. The "normal" X^2 Mandelbrot set is on key 1. (Hitting "1" is a good way to reset yourself if you get lost!) Keys {6 - 0} and {A - N} launch other fractal formulas, which can also be accessed from the "Fractal" menu.

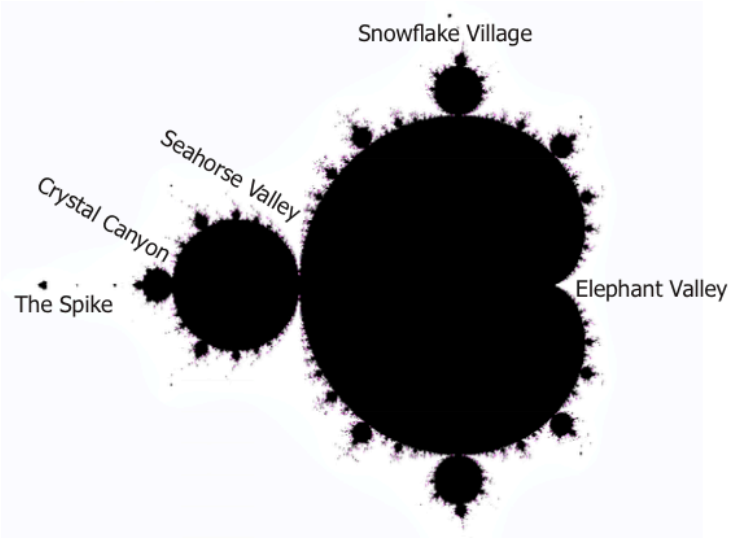
Julia sets: Every point in the Mandelbrot set (and several of the other fractals) corresponds to a unique Julia set. To explore the relationship between the Mandelbrot and Julia fractals, press "J" to enter fast-Julia mode. When you find a Julia set you like, switch over to it by pressing "M".

To save a fractal, there are two ways: "Save Image" saves the image as a PNG file. "Save" will save the parameters in .xpf format (XaoS Parameter File), which allows the fractal to be recreated. To submit fractals to the Fractal Challenge, you must save it BOTH ways.

Finally - use the 'Help' file and explore the excellent tutorials available! They are very useful to learn how to use the program, as well as to learn about the fractals.

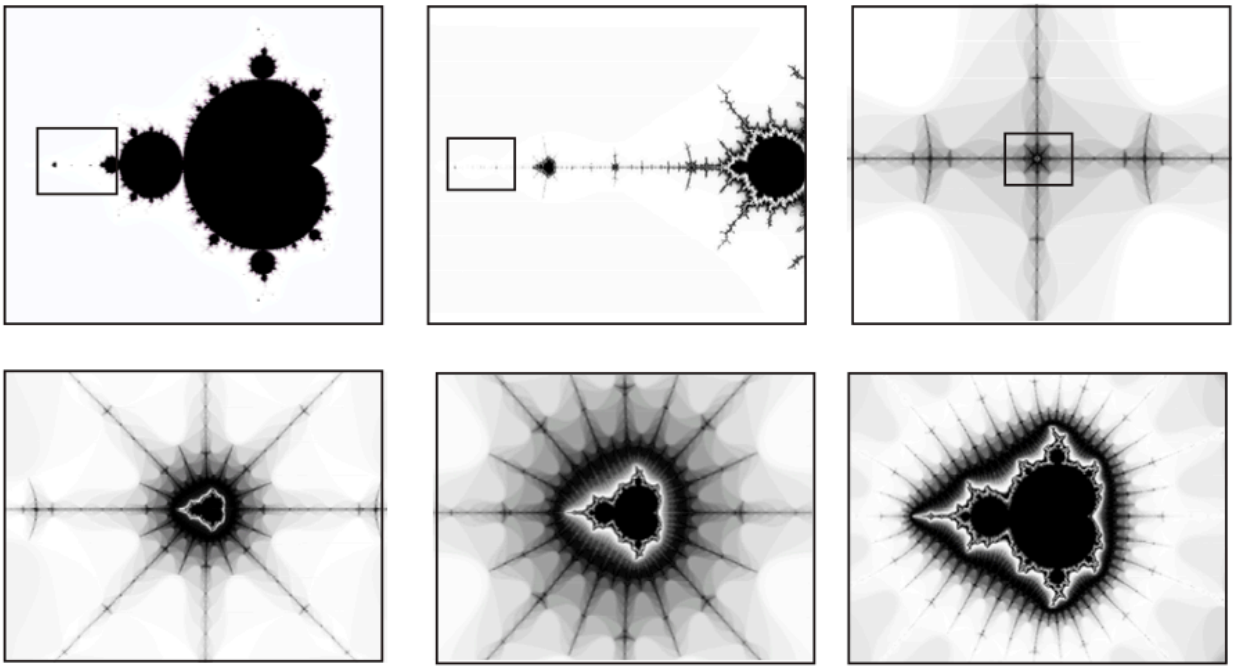
Some observations: First, the most interesting parts are along the edge. Inside the black area there is no detail; outside in the colored area, there is no detail. Along the edge, thereis infinite detail - so focus your exploration there.

As many of the structures along the edge evoke patterns in nature, fractal explorers have given them names to help identify the different regions. See the examples below.

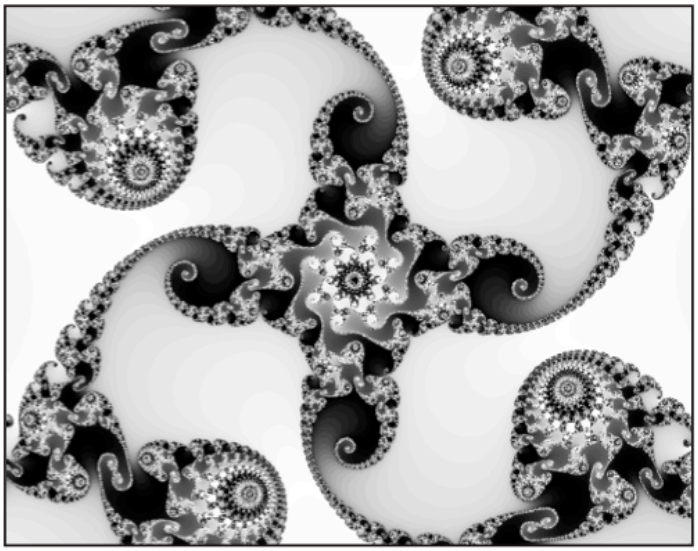


Note that while the structures in the various areas of the Mandelbrot Set are incredibly diverse, the organizational rules are universal. This means that once you learn how the fractal is organized, you can predict the patterns you will observe as you zoom deep into a part of an area you've never explored.

We'll start off by exploring the behavior of the patterns on the "Spike," to the far left of the Mandelbrot Set, because this area is spares and simple. Nonetheless, the rules you will discover here apply everywhere else in the fractal.



Successive zooms into the spike of the Mandelbrot Set show how the symmetry continually doubles. The arms branch in a power of two progression. Starting with 1 axis, or 2-fold symmetry, it turns to 4-fold symmetry, then 8-fold, 16, 32, etc. Finally the arms converge around a tiny copy of the entire Mandelbrot Set.

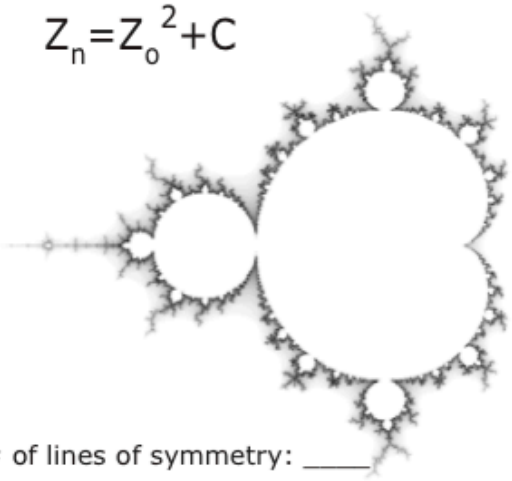


The same doubling progression occurs in every region of the Mandelbrot Set (shown here in the Elephant Valley) with drastically different appearances.

The *symmetry* of a fractal is closely related to its equation. In the images below, draw a line or lines where the fractal is reflected, that is, where are its lines of symmetry?

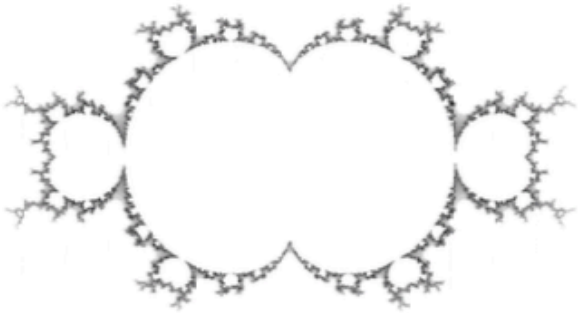
Count how many lines of symmetry there are for each and fill in the number below. Also, measure the angles between each line using a protractor and note them. What trends do you see?

$Z_n=Z_o^{\,2}+C$



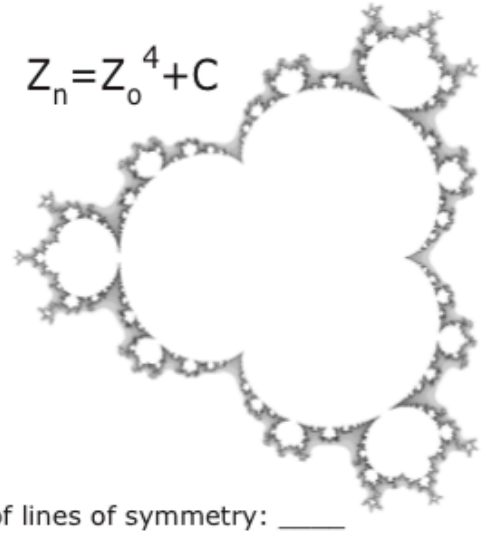
# of lines of symmetry: _____

$Z_n=Z_o^{\,3}+C$



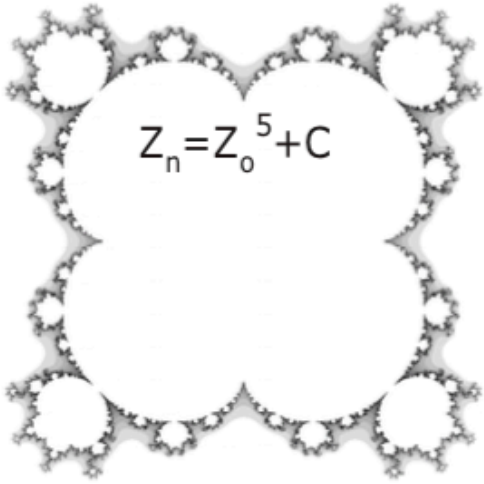
# of lines of symmetry: _____

$Z_n=Z_o^{\,4}+C$



# of lines of symmetry: _____

$Z_n=Z_o^{\,5}+C$



# of lines of symmetry: _____

If you had a fractal made from the equation $Z_n = Z_0^7 + C$, how many lines of symmetry would you predict it would have?

Draw what you think it would look like.