



Name: _____

Fibonacci Sequence and Fractal Spirals

1. First, we're going to figure out the Fibonacci sequence. Fill out the blanks below:

$$0 + 1 = \underline{1}$$

$$1 + \underline{1} = \underline{2}$$

$$\underline{1} + \underline{2} = \underline{3}$$

$$\underline{2} + \underline{3} = \underline{5}$$

$$\underline{3} + \underline{5} = \underline{8}$$

$$\underline{5} + \underline{8} = \underline{13}$$

$$\underline{8} + \underline{13} = \underline{21}$$

$$\underline{13} + \underline{21} = \underline{34}$$

2. List each number after the equal sign: 1 1 2 3 5 8 13 21 34

3. Now, square each number: 1 1 4 9 25 64 169 441

4. Add two adjacent numbers from the list above together.

$$1 + 1 = \underline{2} \quad 1 + 4 = \underline{5} \quad 4 + \underline{9} = \underline{13} \quad \underline{9} + \underline{25} = \underline{34}$$

What pattern do you see? Circle those numbers where you've seen them before!

it's every other number in the Fibonacci sequence

5. How about when you add the squared numbers (from #3) sequentially?

$$1 \quad 1 \quad 4 \quad \underline{9} \quad \underline{25} \quad \underline{64}$$

$1 + 1 + 4 = \underline{6}$ then add the next number in the sequence to that

$$\underline{6} + \underline{9} = \underline{15} + \underline{25} = \underline{40} + \underline{64} = \underline{104}$$

6. List the numbers from above after each equal sign (=): 6 15 40 104

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7. How is each number listed in #6 expressed as a multiplication of numbers in the Fibonacci sequence, listed after #2?

your first number 6 = 2 x 3

your second number 15 = 3 x 5

your third number 40 = 5 x 8

your fourth number 104 = 8 x 13

Another fun and mind-blowing fact...

8. Going back to the original Fibonacci sequence, divide the larger number by the previous smaller number and let's see what we get. The original sequence (#2) is:

1 1 2 3 5 8 13 21 34 55
and so on...

$1 \div 1 = \underline{1}$ $2 \div 1 = \underline{.5}$ $3 \div 2 = \underline{1.5}$ $5 \div 3 = \underline{1.67}$ $8 \div 5 = \underline{1.6}$

$13 \div 8 = \underline{1.625}$ $21 \div 13 = \underline{1.61}$ $34 \div 21 = \underline{1.62}$ $55 \div 34 = \underline{1.62}$

Golden ratio = 1.618033...

9. Let's do some graphing to see more about how this works!

a. What is the first number of the Fibonacci sequence? 1

On the graph paper at the end of this handout, there is square that is 1 x 1.

b. What's the second number of the Fibonacci sequence? 1

Right above the square you just drew, draw another 1 x 1 square.

c. What's the second number in the Fibonacci sequence? 2

Directly to the left of the two existing squares, draw in a 2 x 2 square.

d. What's the next number in the Fibonacci sequence? 3

Right below your existing squares, draw a 3 x 3 square.

e. What's the next number in the Fibonacci sequence? 5

To the right of all that you've drawn, draw a 5 x 5 square.

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f. What's the next number in the Fibonacci sequence? 8
Above all that you've drawn, draw a 8 x 8 square.

g. What's the next number? 13
To the left of all that you've drawn, draw a 13 x 13 square.

h. What's the next number? 21
Below all that you've drawn, draw a 21 x 21 square.

... To the right of that would be the next square, but we've run out of room.

10. Now let's see how we can make a pattern out of these squares.

In the original square, draw a line from the bottom left to the top right.

On the next 1 x 1 square, continue that line across your square, from the bottom right to the top left.

Cross the 2 x 2 square from the top right to bottom left.

Cross the 3 x 3 square from the top left to bottom right.

Cross the 5 x 5 square from bottom left to top right.

Cross the 8 x 8 square from bottom right to top left.

Continue the line across the 13 x 13 square and the 21 x 21 square, wrapping up with a line that would go through the 34 x 34 square.

11. What pattern do you get?

Spiral

12. Where do we find spirals naturally?

*snail, tornado, hurricane, spiral galaxy, broccoli
Romanesco, pineapple, pinecone, sunflower seeds, etc*

13. Count the number of things that make up a spiral on a pineapple or a pine cone or the number of petals on a flower or number of spirals on a broccoli or seeds of a sunflower.

They all occur in Fibonacci numbers! Nature is full of mathematical patterns! Amazing, huh? See what other cool patterns you can figure out in nature.

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